PILOT

# Problem Set 10

Differential Equations

Fall 2024

Up to now, we have finished the first order linear system, and we are now sneaking into nonlinear systems. As we get into the chapter, let us review what we have learned about linear systems.

- Concepts:
  - Vector space
  - Eigenspace
  - Existence & Uniqueness Theorem
- Methods to solve linear systems of ODEs:
  - Eigenvectors & Eigenvalues
  - Repeated Roots
  - Phase Portraits and Stability

1. (Stability for Nonlinear System). Complete the following table for stability of dimension 2 linear and nonlinear systems.

| **Eigenvalues** | **Linear System** | | **Nonlinear System** | |
|---|---|---|---|---|
| | **Type** | **Stability** | **Type** | **Stability** |
| Eigenvalues are $\lambda_1$ and $\lambda_2$ | | | | |
| $0 < \lambda_1 < \lambda_2$ | | | | |
| $\lambda_1 < \lambda_2 < 0$ | | | | |
| $\lambda_1 < 0 < \lambda_2$ | | | | |
| $\lambda_1 = \lambda_2 > 0$ | | | | |
| $\lambda_1 = \lambda_2 < 0$ | | | | |
| Eigenvalues are $\lambda_1 = \alpha + i\beta$ and $\lambda_2 = \alpha - i\beta$ | | | | |
| $\alpha > 0$ | | | | |
| $\alpha = 0$ | | | | |
| $\alpha < 0$ | | | | |

2. (Phase Portraits for Repeated Roots). Find the solutions to the following linear system differential equation, sketch a few phase portraits, and classify its type and stability.

   (a) $$\mathbf{x}' = \begin{pmatrix} 4 & 2 \\ -2 & 0 \end{pmatrix} \cdot \mathbf{x}.$$

   (b) $$\mathbf{x}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \cdot \mathbf{x}.$$

3. (Critical Point). Find all the critical point in the following first order system:
$$\begin{cases} x' = 2x^3 - x^2 - 4x + 3 - y^2, \\ y' = 2x - y. \end{cases}$$

4. (Locally Linear System). Let a linear system be defined as follows:
$$\begin{cases} x' = x - y, \\ y' = x - 2y + x^2. \end{cases}$$
   (a) Verify that $(0,0)$ is an equilibrium.
   (b) Verify that the system is locally linear at $(0,0)$.
   (c) Classify the type and stability of $(0,0)$ locally.

**Clubs & Orgs Bulletin**

Promote your club! `https://forms.gle/V19BipzLyuAaWMyz8`

**Behavioral Biology Steering Committee:** *Want to explore about a career in veterinary medicine? Join the Behavioral Biology committee this Friday at 5:30 for a Pre-Vet meetup to discuss the path and opportunities. Stick around until 6:30 for a special showing of "Back to the Future!"*

**Witness Theater:** *Our next production is 24-Hour Show (11/8-11/9)! This is the perfect opportunity for anyone looking to try something newwhether it's acting, directing, tech, or writing. It's a fun, low-commitment experience with no experience required to be involved! Learn more @ linktr.ee/witnesstheater*

**Tip of the Week**

*The Red Zone refers to the period of time between the start of fall semester and Thanksgiving when the majority of sexual assaults on campus occur. The peak in the Red Zone occurs around Halloween, so it is important to refresh your knowledge on consent and sexual assault resources during this time. Confidential sexual assault resources at JHU include the Counseling Center Sexual Assault Helpline at (410) 516-7333 and the Hopkins Sexual Assault Resource Unit Hotline at (410) 516-7887. Access more resources and educational materials on consent on Instagram @jhusaru and @chewatjhu.*