



Problem Set 11

Differential Equations

Fall 2024

In the digressions to non-linear system, we are gradually seeing more dynamical systems and applications to the real life situations. As we explore cases with the non-linear system, please keep in mind that we are always trying to find a linear case to model as we zoom in.

1. (System with Unknown Coefficients). Let a non-linear system for $x = x(t)$ and $y = y(t)$ be:

$$\begin{cases} x' = \alpha x - y + y^2, \\ y' = x + \alpha y. \end{cases}$$

- (a) Show that $(0,0)$ is a critical point, and show system is locally linear at $(0,0)$ for all $\alpha \in \mathbb{R}$.
(b) Classify the critical point $(0,0)$ and sketch a few phase portraits of the linearized system.

2. (Nonlinear at origin). Let the linear system be:

$$\begin{cases} x' = y, \\ y' = x + 2x^3. \end{cases}$$

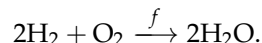
- (a) Show that the origin is a saddle point.
(b) Sketch a phase portrait for the linearized system. Note that where all the trajectories of the linear system tend to the origin.

3. (Modeling Politics). Suppose D and R are two parties on a non-existing country on the center of Mars. For the simplicity of this problem, they, *unfortunately*, have no elections. Therefore, we can model the amount of the supporter for each party (in millions), denoted x_D and x_R with the following relationship:

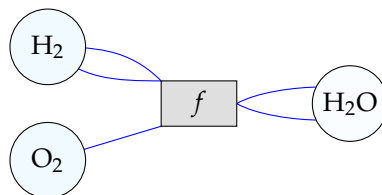
$$\begin{cases} \frac{dx_D}{dt} = x_D(1 - x_D - x_R), \\ \frac{dx_R}{dt} = x_R(3 - 2x_D - 4x_R). \end{cases}$$

Find all possible endings (say arbitrarily long after, that is $t \rightarrow \infty$) of the number of supporters (in millions) for the two parties.

4. (Chemical Reaction). Consider the following chemical equation of hydrogen gas combustion in oxygen gas:



We may represent it from a graphical representation.



Assume that the reaction rate is constant $\kappa := \text{rate}(f)$. Construct the nonlinear system of the concentration of H_2 and O_2 , sketch a few trajectories for different initial conditions for different starting concentrations.

Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

JHU Climbing Wall & Bouldering Cave: *Feeling boulder this semester? Harness your inner climber and hang around the climbing wall! The wall and cave are open daily for beginners and experienced climbers alike with belay classes and event nights. Need climbing buddies? Join Rockhoppers club! More info on our Instagram @jhucimbingwall.*

Diverse Sexuality and Gender Alliance (DSAGA): *Join us every Monday 8:15-9:15 PM in Maryland 217 to spend time with a wonderful community of students who identify as members of the LGBTQ+ community. You will meet other students, plan activities, and engage with the community. <https://linktr.ee/jhuDSAGA>*

Tip of the Week

Looking for off-campus housing for the summer/fall? Check out the Johns Hopkins Off-Campus website which offers resources like apartment listings and roommate matches: <https://offcampushousing.jhu.edu/>. Don't feel like paying rent? Consider becoming an RA! Info sessions will be hosted throughout the next couple of months both in-person and virtually. Find out more @jhureslife on IG.