P LOT

Problem Set 2

Differential Equations

Fall 2024

Now, as we have dived into the contents of first order linear differential equations, you should have noticed many techniques, such as method of separation, integrating factors, or exactness. Meanwhile, we are about the see how these methods can be applied into various system, of how we can model by ODEs.

1. (Linearity of Solutions.) Let $y = y_1(t)$ be a solution to $y' + p(t)y = 0$, and let $y = y_2(t)$ be a solution to $y' + p(t)y = q(t)$. Show that $y = y_1(t) + y_2(t)$ is then also a solution to $y' + p(t)y = q(t)$.

2. (Integrating Factor.) Solve for the general solution to the following ODE with $y = y(t)$:
$$2y' + y = 3t.$$

3. (Integrating Factor or Exactness?) Let a differential equation be defined as follows:
$$\frac{dy}{dx} = e^{2x} + y - 1.$$

   (a) What is the integrating factor ($\mu(x)$) for the equation? Solve for the general solution.

   (b) Is the equation *exact*? If not, make it exact, then find the general solution.

   (c) Do solutions from part (a) and (b) agree?

4. (Decay and Dating.) Carbon-14, a radioactive isotope of carbon, is an effective tool in dating the age of organic compounds, as it decays with a relatively long period. Let $Q(t)$ denote the amount of carbon-14 at time t, we suppose that the decay of $Q(t)$ satisfies the following differential equation:
$$\frac{dQ}{dt} = -\lambda Q \text{ where } \lambda \text{ is the rate of decay constant.}$$

   (a) Let the half-life of carbon-14 be τ, find the rate of decay, λ.

   (b) Suppose that a piece of remain is discovered to have 10% of the original amount of carbon-14, find the age of the remain in terms of τ.

<div style="border:1px solid">

Clubs & Orgs Bulletin

Promote your club! `https://forms.gle/V19BipzLyuAaWMyz8`

Matriculate: *Matriculate is a national organization that allows college students to give admissions guidance to high-achieving, low-income high schoolers across the US. JHU Matriculate advisors are paid. Applications are accepted through 9/23 - visit* `http://matriculate.org/become-an-advising-fellow/` *for more!*

bARTimore: *Our purpose is to bring Baltimore inspired art onto and near campus in an effort to pop the 'Hopkins Bubble'. We help students engage with the community by painting murals and hosting art events emphasizing Baltimorean themes. If you're interested, check us out on IG: @bartimoreathopkins!*

Youth Library Tutorial: *Are you interested in tutoring? Youth Library Tutorial (YLT) is a community service organization that provides one-on-one K-12 tutoring to students in the Baltimore community. If you want to learn more, please visit our Instagram @hopkinsylt for more information.*

</div>

PILOT Tip of the Week

Ready to see the world? The Global Education Office has resources for Study Abroad programs in over 50 countries worldwide, for all areas of study. They provide help with applications, scholarships, and accommodations. Intersession and Spring semester application deadlines are coming up, so find out more at `https://studyabroad.jhu.edu/`*!*