P✈LOT

Problem Set 3

Differential Equations

Fall 2024

We have finished with contents in foundations and first order linear ODEs. As we wrap up, let's remark on the key components so far in the class:

- Concepts:
    - Existence
    - Uniqueness
    - General Solutions
    - Specific Solutions (IVPs)

- Methods to solve first order ODEs:
    - Separable ODEs
    - Integrating Factor
    - Exact ODEs
    - Autonomous ODEs

- Behavior analyses:
    - Directional Field
    - End Behavior
    - Phase Line
    - Bifurcation Diagram

As we are about to tackle ODEs with higher order, let us consolidate what up to first order.

1. (Stability.) Draw the phase line and determine the stability of each equilibrium:
$$y' = y^2(y-1)(y-2).$$

2. (Ending Behavior.) Given an IVP as follows:
$$\begin{cases} y' + \dfrac{1}{2}y = \sin t; \\ y(0) = 1. \end{cases}$$
    Find the specific solution of the equation and describe its end behavior

3. (Bifurcation.) For the first-order autonomous ODE:
$$\frac{dx}{dt} = x^2 - 2x + c,$$
    with parameter $c \in \mathbb{R}$, do the following:

    (a) Sketch all of the qualitatively different graphs of $f(x) = x^2 - 2x + c$, as c is varied.

    (b) Determine any and all bifurcation values for the parameter c.

    (c) Sketch a bifurcation diagram for this ODE.

4. (Existence of Higher Order ODEs.) Determine intervals that the initial condition must be in so that the solutions are sure to exist:

    (a) $y^{(4)} + 4y''' + 3y = t$,

    (b) $y''' + ty'' + t^2 y' + t^3 y = \ln(t)$.

Clubs & Orgs Bulletin

Promote your club! https://forms.gle/V19BipzLyuAaWMyz8

Bands@JHU: *Do you play a band instrument and want to be involved in an ensemble? Bands@JHU is the place to be! Rehearsals are Sunday evenings for the wind ensemble and Thursday evenings for the jazz bands. We welcome all musicians wanting to make some good music, so check us out at* `https://tinyurl.com/ycz8zupt`.

Barnstormers Theater Group: *Interested in theater? Join the Barnstormers' production of Twelfth Night this fall! Tech applications are open until September 22, and auditions will be on September 20 and 21. Apply for a tech role or audition slot here:* `https://linktr.ee/jhubarnstormers`.

A Place to Talk: *A Place to Talk is a peer listening group that lets anyone discuss anything on their mind. Come to Brody 4010 or Wolman where listeners will help you explore feelings without giving advice. Check our instagram @jhuaptt &* `https://pages.jh.edu/aptt/index.html` *for more info & our listener application!*

Tip of the Week

Have you used Handshake to find jobs and internships at or beyond Hopkins? Handshake offers career workshops, recruitment information sessions, and job opportunities that target Johns Hopkins students and alumni. Setting up your Handshake profile can also make you visible to more recruiters and employers. It is a great professional resource designed to connect students with opportunities in their fields of interest: `https://jhu.joinhandshake.com/login`.