



Problem Set 6

Differential Equations

Fall 2024

As of right now, we have completed our expenditure of higher order differential equations. You should be familiar with the following concepts:

- Concepts:
 - Set of Solutions
 - Linear Independence
 - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
 - Characteristic Equation
 - Euler's Formula
 - Undetermined Coefficients
 - Reduction of Order
 - Variation of Parameters

Now, as we step into more linear algebra, we are going to review the key contents of this part of the class.

1. (Reduction of Order or Integrating Method). Let a differential equation be:

$$y''(t) + \frac{2}{t}y'(t) = 0.$$

- (a) Verify that $y(t) = 1/t$ is one solution, then find a full set of solution.
- (b) Consider $\omega(t) = y'(t)$, solve the differential equation by using integrating factor.
- (c) Verify that the two methods give you the same set of the solutions.

2. (Complex Characteristics, Again). Find a full set of real solutions to the differential equation:

$$\frac{d^3y}{dx^3} = -y.$$

3. (Non-homogeneous Differential Equations). Solve the following differential equations.

(a) $y'' + 4y = t^2 + 3e^t.$

(b) $y'' + 2y' + y = \frac{e^{-x}}{x}.$

4. (Warm up in Linear Algebra). This problem reviews the basic concepts linear algebra concepts.
- (a) Which of the following set of vectors are linearly independent in \mathbb{R} -vector space, what about \mathbb{C} -vector space? Justify your answer.
- (i) $\alpha = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\},$
- (ii) $\beta = \{(0, 1), (2, 3), (4, 5)\},$
- (iii) $\gamma = \{1, i\}.$
- (b) Let $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$ and $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$, compute the following:
- (i) $A - 2B,$
- (ii) $BA,$
- (iii) $B^{-1}.$

Note: Change in PILOT Schedule Next Week

Fall break is the next week. Sessions on next Thursday (10/17) and Friday (10/18) will be canceled. Please refer to the **Problem Set 7** for the problems in advance and the virtual PILOT session information. Have a nice fall break!

Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

Hopkins Community Connection: Have you heard of Hopkins Community Connection? HCC is a student-supported organization supervised by JHU staff that connects patients to essential community resources. Hopkins Groups link <https://jhu.campusgroups.com/hcc/home/> where you can find the application form. App. deadline is 10/15/2024

Tip of the Week

Student Outreach & Support (SOS) provides one-on-one support for students through meetings, advocacy, referrals to resources, and follow-up services. These include Emergency & Medical Leave of Absence, Emergency Fund, and the Laptop Loaner Program. Visit <https://studentaffairs.jhu.edu/sos> for more.