Problem Set 8

Differential Equations

Fall 2024

Welcome back from the Fall Break. We hope that you have had a chance to relax yourself. For this week, we will explore deeper into the linear systems as well as special cases in linear system, *i.e.*, complex or zero eigenvalues. Alias, lets explore further on what eigenvalues represent and the geometries behind it.

1. (Linear System versus Second Order). Let an initial value problem for linear system on $x_1 := x_1(t)$ and $x_2 := x_2(t)$ be defined as follows:

$$\begin{cases} x_1' = 3x_1 - 2x_2, & x_1(0) = 3, \\ x_2' = 2x_1 - 2x_2, & x_2(0) = \dfrac{1}{2}. \end{cases}$$

   (a) Solve for the *general solution* for the linear system by considering $\mathbf{x} = (x_1, x_2)$.

   (b) Transform the *general system* into a single equation of second order. Then solve the second-order equation. Eventually, convert your solution of one variable back to the *general solution* to $x_1(t)$ and $x_2(t)$.

   (c) Find the particular solution using the initial conditions, then graph the parameterized curve on a x_1x_2-plane with $t \geq 0$.

2. (Deal with Complex Eigenvalues). Let $\mathbf{x} = (x_1, x_2, x_3)$ be in dimension 3, we define a linear system as follows:

$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} .\mathbf{x}.$$

   Find the general solution to the above system in terms of real valued functions.

3. (Directional Field for Linear System). For the following systems with $\mathbf{x} = (x_1, x_2)$, draw a direction field and plot some trajectories to characterize the solutions.

   (a) $$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -2 & 2 \end{pmatrix} .\mathbf{x}.$$

   (b) $$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} .\mathbf{x}.$$

4. (Zero Eigenvalue). Let a system of $\mathbf{x} = (x_1, x_2)$ be defined as:

$$\mathbf{x}' = \begin{pmatrix} -3 & -6 \\ 1 & 2 \end{pmatrix} .\mathbf{x}.$$

   (a) Find the eigenvalues and eigenvectors for $\begin{pmatrix} -3 & -6 \\ 1 & 2 \end{pmatrix}$.

   (b) Give a full set of solutions to the differential equation. Plot some trajectory on the x_1x_2-plane.

   (c)* Let A be an arbitrary square matrix. Show that A is non-invertible if and only if A has zero as an eigenvalue.
   Note: Please avoid using the definition that the determinant is the product of all eigenvalues. Moreover, consider the geometric implication of eigenvalue to account for invertibility.

Clubs & Orgs Bulletin

Promote your club! `https://forms.gle/V19BipzLyuAaWMyz8`

Johns Hopkins American Chemical Society (JHACS): *Come by the Remsen Breezeway for some nitrogen ice cream, freshly made by the American Chemical Society on 10/23/24 from 10:30am-12:00pm. Learn about our upcoming events, community outreach, and how you can get involved! Follow us on instagram for more information @johnshopkinsacs*

Sexual Assault Resource Unit (SARU): *Join SARU x FibreArts on Oct 28th for a Halloween event, where we raise awareness about consent, while enjoying some spooky fun! We'll have a costume contest with exciting prizes, including candy and sex toys! There will also be activities and free food!*

Tip of the Week

As we approach Election Day on November 5thjust THREE weeks awayit's essential to make sure you are registered and ready! If you haven't registered to vote yet or need help requesting an absentee ballot, please visit https://jhu.turbovote.org! While casting your vote is incredibly important, there are many other ways to engage in civic life. You can volunteer with local organizations, attend town hall meetings, or help educate your peers about the issues that are important to you. If you need further assistance, please reach out to Hopkins Votes at https://hopkinsvotes.jhu.edu/