

From this week, we are gradually transforming from linear system into non-linear system. As we generalize the cases of a linear system, always remind yourself with the notion of eigenspace and the geometries behind it. For non-linear system, you shall be thinking of how to remake into a linear system that we are familiar with.

1. (Repeated Eigenvalue). This problem investigates the case for repeated eigenvalues. First, we let the matrix $A \in \mathbb{R}^{2 \times 2}$ be:

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}.$$

Here, we define the *algebraic multiplicity* of an eigenvalue as its multiplicity as a root to the characteristic polynomial, and the *geometric multiplicity* is the dimension of the eigenspace.

- (a) Find the eigenvalue and its corresponding eigenvector. State its algebraic and geometric multiplicity.
- (b) Find a the general solution to $\mathbf{x}' = A \cdot \mathbf{x}$, where $\mathbf{x}' \in \mathbb{R}^2$.

Then, we consider the diagonal *n*-by-*n* matrices, that is matrices with entries only on the diagonal, which can be characterized as:

$$D = \begin{pmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_n \end{pmatrix} \in \mathbb{R}^{n \times n}$$

- (c) Show that the eigenvalues are exactly a_1, \dots, a_n , and the algebraic multiplicity is exactly the same as geometric multiplicity for all eigenvalues.
- (d) Consider the linear system $\mathbf{x} = D.\mathbf{x}$ for $\mathbf{x} \in \mathbb{R}^n$, solve for the general solution for $\mathbf{x} = (x_1, \dots, x_n)$. Explain why do not have to find the eigenvalues in this case.
- 2. (Complex Eigenvalue and Phase Portraits). Find the general solution and sketch a few phase portraits for:

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} .\mathbf{x}.$$

Eigenvalues	Туре	Stability
Eigenvalues are λ_1 and λ_2		
$0 < \lambda_1 < \lambda_2$		
$\lambda_1 < \lambda_2 < 0$		
$\lambda_1 < 0 < \lambda_2$		
$\lambda_1 = \lambda_2 > 0$		
$\lambda_1 = \lambda_2 < 0$		
Eigenvalues are $\lambda_1 = \alpha + i\beta$ and $\lambda_2 = \alpha - i\beta$		
$\alpha > 0$		
$\alpha = 0$		
$\alpha < 0$		

3. (Stability). Complete the following table for stability of dimension 2 linear systems.

4. (Fundamental Matrix). Let a system be defined as:

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}.$$

Find the general solution using a fundamental matrix.

Clubs & Orgs Bulletin

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Futurism at JHU: We are calling all science and technology enthusiasts! Our podcast, Futurism at JHU, explores the future of innovation by discussing the latest discoveries, ranging from biotech to AI startups, and interviewing guests like President Daniels! Join our club today: https://forms.gle/jQ6sinb6uqYcY96bA

Johns Hopkins Badminton Tournament: The Johns Hopkins Badminton Club is hosting a Doubles Tournament on Nov 16-17! The event will follow a bracket system so players of all skill levels are guaranteed competitive matches. Register to play: http://bit.ly/3YOEVIQ by Nov 8! Find us on Instagram @jhubadminton (registration link also here)

Tip of the Week

Love thrifting, crafts, and good food? The Bmore Flea Market is open every Saturday until December from 10am-3pm at Broadway Market in Fells Point. Check out their Instagram @bmoreflea for more info!