



## Problem Set 11

### Differential Equations

Fall 2025

In the digressions to non-linear system, we are gradually seeing more dynamical systems and applications to the real life situations. As we explore cases with the non-linear system, please keep in mind that we are always trying to find a linear case to model as we zoom in. At the same moment, please take a moment to review what we have learned together.

- Concepts:
  - Nonlinear System
  - Locally Linear System
  - Jacobian Matrix
- Models:
  - Predator-Prey Model
  - Competing Species Model
  - Limit Cycles\*

### Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

**Motorsports Society:** Are you interested in motorsport, whether you're an F1 fan, becoming a mechanical engineer, or pursuing a career in sports journalism? Join Motorsoc! We foster a fun, inclusive environment for all kinds of motorsport fans. Follow our Instagram @motor\_soc\_jhu for more updates!

**Maryland Science Olympiad:** Did you do Science Olympiad or similar competitions in high school? Are you interested in helping organize, volunteer, and write for Maryland Science Olympiad's regional and state tournaments? Join the Maryland Science Olympiad @ JHU chapter! Sign up: [https://jhu.campusgroups.com/MSO/club\\_signup](https://jhu.campusgroups.com/MSO/club_signup).

### Tip of the Week

*Study with the help of a Learning Den Tutor? Build a finals study schedule with a study consultant? CSS Saturday is on Nov. 15, Dec. 6, and Dec 13 at 12–4pm.*

*Course supported: Introductory Chemistry I, Organic Chemistry I, Physics I & II, PreCalculus, Calculus I, II, II, Gateway Computing: Java, Python.*

1. (System with Unknown Coefficients). Let a non-linear system for  $x = x(t)$  and  $y = y(t)$  be:

$$\begin{cases} x' = \alpha x - y + y^2, \\ y' = x + \alpha y. \end{cases}$$

- (a) Show that  $(0,0)$  is a critical point, and show system is locally linear at  $(0,0)$  for all  $\alpha \in \mathbb{R}$ .  
(b) Classify the critical point  $(0,0)$  and sketch a few phase portraits of the linearized system.

2. (Critical Point). Find all the critical point in the following first order system:

$$\begin{cases} x' = 2x^3 - x^2 - 4x + 3 - y^2, \\ y' = 2x - y. \end{cases}$$

3. (Nonlinear at origin). Let the linear system be:

$$\begin{cases} x' = y, \\ y' = x + 2x^3. \end{cases}$$

- (a) Show that the origin is a saddle point.
- (b) Sketch a phase portrait for the linearized system. Note that where all the trajectories of the linear system tend to the origin.

4. (Modeling Politics). Suppose  $D$  and  $R$  are two parties on a non-existing country on the center of Mars. For the simplicity of this problem, they, *unfortunately*, have no elections. Therefore, we can model the amount of the supporter for each party (in millions), denoted  $x_D$  and  $x_R$  with the following relationship:

$$\begin{cases} \frac{dx_D}{dt} = x_D(1 - x_D - x_R), \\ \frac{dx_R}{dt} = x_R(3 - 2x_D - 4x_R). \end{cases}$$

Find all possible endings (say arbitrarily long after, that is  $t \rightarrow \infty$ ) of the number of supporters (in millions) for the two parties.