Problem Set 12

Differential Equations

Fall 2025

Attention on Different Contents.

The questions labeled with **S** are the questions designated for Series Solutions for Second Order ODEs (Section 01-03) and the questions labeled with **L** are the questions designated for Laplace Transformation (Section 04-06).

Welcome to the final chapters of the course. Depending on your choice, you will be exploring about one of the two methods to find the solutions to specific types of differential equations. Series solutions allows you to consider differential equations as a sequence of polynomials, whereas Laplace transformation lifts the solutions to another space so we can map our solutions correspondingly.

Clubs & Orgs Bulletin

Promote your club! https://forms.gle/V19BipzLyuAaWMyz8

HopTHON: Are you interested in raising money for children in need? We work with Johns Hopkins Childrens Center under Childrens Miracle Network of Hospitals to raise money for the families and children undergoing treatment! We have many fun ways to get involved and fundraising events throughout the year.
A Place To Talk: Need to vent about something or talk through an issue? Come visit an APTT room! Want to encourage your organizations' members to be more compassionate and welcoming? Schedule listening and empathy trainings by emailing apttexternaltraning@gmail.com. Learn more: `@jhuaptt` or https://pages.jh.edu/aptt/.

Tip of the Week

Looking for off-campus housing for the summer/fall? Check out the Johns Hopkins Off-Campus website which offers resources like apartment listings and roommate matches: https://offcampushousing.jhu.edu/. Furthermoreconsider becoming an RA! Info sessions will be happening through the next couple of months both in-person and virtually. Find out more `@jhureslife` *on ig.*

1. (Limit Cycles). Determine the periodic solution, if there are any, of the following system:

$$\begin{cases} x' = y + \dfrac{x}{\sqrt{x^2+y^2}}(x^2+y^2-2), \\ y' = -x + \dfrac{y}{\sqrt{x^2+y^2}}(x^2+y^2-2). \end{cases}$$

S2. (Converging Sequences). In this question, we will review some common power series.

(a) Construct the power series of e^x, $\sin x$, and $\cos x$ centered at 0.

(b) Consider the following power series:

$$\sum_{k=0}^{\infty} \frac{x^{4k+3}}{(4k+3)!}.$$

Identify if such series converges. Compute the limit if the series converges.

L2. (Laplace Transformation). Show the following Laplace transformation by definition.

(a) $$\mathcal{L}\{\sin(at)\} = \frac{a}{a^2 + s^2}.$$

(b) $$\mathcal{L}\{(f * g)(t)\} = \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\}.$$

S3. (First Order Recurrence Relationship). For this problem, we will be solving a first order differential equation using the series solution.

(a) Deduce the power series for $\log(x+1)$ centered at 0. What is the interval of convergence for the power series?

(b) Consider the differential equation given by:

$$(x+1)y'(x) = 1,$$

find the recurrence relationship and find the solution to the differential equations.

(c) Solve the differential equation using the separation of variables and compare your answer with the previous part.

L3. (Convolution versus Laplace). Consider the function $f(t) = t^2$ and $g(t) = \cos t$.

(a) Compute $(f * g)(t)$ by the definition of convolution.

(b) Compute $(f * g)(t)$ using Laplace transformation formula.

S4. (Recurrence Relation). Solve the following differential equation using power series method. Include the recurrence relation.

$$y'' + y = 0.$$

L4. (Solving IVP with Laplace) Solve the following IVP using the Laplace transform:

$$\begin{cases} y'' - 10y' + 9y = 5t, \\ y(0) = -1, \qquad u'(0) = 2. \end{cases}$$