PILOT

Problem Set 3

Differential Equations

Fall 2025

Welcome back to more ODEs. As we are familiarizing ourselves with the techniques of solving ODEs, let's practice our basics skills while also delve into some theory-intensive contents. It is important to acquaint yourself with the skills to solve equations quickly while also equip yourself with knowledge of how things work deep in the box.

Tip of the Week

The Hopkins Food Pantry is a free resource for JHU affiliates facing food insecurity. Located at the LaB (right next to Homewood Apartments), it's open weekly on Mondays and Tuesdays to registered shoppers.
Learn more at https://studentaffairs.jhu.edu/student-life/student-outreach-support/hopkins-food-pantry/.

1. (Stability of Autonomous ODEs). Draw the phase line and determine the stability of each equilibrium for the following autonomous differential equations:

   (a) $$y' = y^4 - 3y^3 + 2y^2.$$

   (b) $$y' = y^{2025} - 1.$$

   (c) $$y' = y^2 + 2y + C, \text{ where } C \in \mathbb{R} \text{ is a constant.}$$

   For part (c), determine the bifurcation values for the parameter C and sketch a bifurcation diagram.

2. (More IVPs). Given following IVPs:

   (a)
$$\begin{cases} y' + \dfrac{1}{2}y = \sin t, \\ y(0) = 1. \end{cases}$$

   (b)
$$\begin{cases} y' = \dfrac{1}{x^4 - 1}, \\ y(0) = 0. \end{cases}$$

   Find the specific solution of the equation, note the domain of the solution, and describe the end behavior of the solution if it appears in the valid domain.

3. (Existence and Uniqueness for IVP). Suppose $f(x)$ is non-zero, let an initial value problem be:

$$\begin{cases} \dfrac{1-y}{x} \cdot \dfrac{dy}{dx} = \dfrac{f(x)}{1+y}, \\ y(0) = 0. \end{cases}$$

(a) Show that the differential equation is **not** linear.

For the next two parts, suppose $f(x) = \tan x$.

(b) State, without justification, the open interval(s) in which $f(x)$ is continuous.

(c) Show that there exists some $\delta > 0$ such that there exists a unique solution $y(x)$ for $x \in (-\delta, \delta)$.

Now, suppose that $f(x)$ is some function, **not** necessarily continuous.

(d) Suppose that the condition in (c) does **not** hold, give three examples in which $f(x)$ could be.

4. (Existence of Largest Interval). For the following IVPs, determine the largest interval in which a solution is guaranteed to exist.

(a)
$$\begin{cases} (t-3)y' + (\log t)y = 2t, \\ y(1) = 2. \end{cases}$$

(b)
$$\begin{cases} (4-t^2)y' + 2ty = 3t^2, \\ y(1) = -3. \end{cases}$$

(c)
$$\begin{cases} y' + (\tan t)y = \sin t, \\ y(\pi) = 0. \end{cases}$$