PILOT

Problem Set 4

Differential Equations

Fall 2025

As we wrap up the first main part of the class, let's remark on the key components so far about first order ordinary differential equations:

- Concepts:
  - Existence
  - Uniqueness
  - General Solutions
  - Specific Solutions (IVPs)
- Methods to solve first order ODEs:
  - Separable ODEs
  - Integrating Factor
  - Autonomous ODEs
  - Exact ODEs
- Behavior analyses:
  - Directional Field
  - End Behavior
  - Phase Line
  - Bifurcation Diagram

Get ready for more classes of differential equations!

Tip of the Week

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1. (Make an Exact ODE). Let a differential equation on $y := y(x)$ be defined as follows:

$$xy^2 + bx^2y + (x+y)x^2y' = 0.$$

Suppose this differential equation is exact. Find the appropriate value of b and then solve for the solution of the differential equation.

2. (Bifurcation Diagram). For the first-order autonomous ODE:

$$\frac{dx}{dt} = x^2 - 2x + c,$$

with parameter $c \in \mathbb{R}$, do the following:

(a) Sketch all of the qualitatively different graphs of $f(x) = x^2 - 2x + c$, as c is varied.

(b) Determine any and all bifurcation values for the parameter c.

(c) Sketch a bifurcation diagram for this ODE.

3. (Integrating Factor & Exactness). Let a differential equation be defined as follows:

$$\frac{dy}{dx} = e^{2x} + y - 1.$$

   (a) What is the integrating factor ($\mu(x)$) for the equation? Solve for the general solution.

   (b) Is the equation *exact*? If not, make it exact, then find the general solution.

   (c) Do solutions from part (a) and (b) agree?

4. (Preliminary to Second Order ODEs). Let a second order differential equation be defined as follows:

$$y'' - 2y' + y = 0.$$

   (a) Verify that $y_1 = e^t$ and $y_2 = te^t$ are two solutions to the above differential equation.

   (b) Verify that any *linear combination* of y_1 and y_2 is a solution to the above differential equation.