



## Problem Set 6

### Differential Equations

Fall 2025

As we delve deeper into the study of second order differential equations, we will see how our concepts can be extended to more classes of differential equations. We will think of the main components of second order differential equations, explore the reduction of order as well as understand linear independence through a conceptual level.

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#### Tip of the Week

*The beginning of fall also means the start of flu season—get your shot today! Hopkins holds flu clinics on all of its campuses. Students must upload verification of their immunization or a valid exemption by Friday, Nov 21st, 2025. Find out more information here: <https://wellbeing.jhu.edu/PrimaryCare/annual-flu-vaccine-requirement/>.*

1. (Constructing Solutions, Again.) Construct an initial value problem with Cauchy conditions for the following solutions:

(a)  $y(t) = 4e^{3t} - e^{-2t}.$

(b)  $y(t) = e^{2t} \cos t + e^{2t} \sin t + e^{2t}.$

2. (Reduction of Order). Let a ODE be defined as follows:

$$t^2y'' + 2ty' = 2y, t > 0.$$

Given a solution is  $y_1(t) = t$ , find the other solution by assuming  $y_2(t) = u(t) \cdot y_1(t)$ .

3. (Reduction of Order or Integrating Method). Let a differential equation be:

$$y''(t) + \frac{2}{t}y'(t) = 0.$$

- (a) Verify that  $y(t) = 1/t$  is one solution, then find a full set of solution.
- (b) Consider  $\omega(t) = y'(t)$ , solve the differential equation by using integrating factor.
- (c) Verify that the two methods give you the same set of the solutions.

4. (A Criterion on Linearity Independence). Recall that for the complex characteristic roots  $x = \lambda \pm i\mu$ , the corresponding solutions are:

$$y_1 = e^{\lambda x} \sin(\mu x) \quad \text{and} \quad y_2 = e^{\lambda x} \cos(\mu x).$$

Of course, you may compute the Wronskian of  $W(e^{\lambda x} \sin(\mu x), e^{\lambda x} \cos(\mu x))$ , which involves taking derivative with chain rule and product rule, and a lot of computation. Another approach to show linear independence is by returning to its definition.

**Definition.** (Linearity Independence).

Two functions  $f$  and  $g$  are *linearly independent* if  $\lambda_1 f + \lambda_2 g = 0$  implies  $\lambda_1 = \lambda_2 = 0$ .

- (a) Show that  $y_1 = x$  and  $y_2 = x^2$  is linearly independent via both approach.
- (b) Show that  $y_1 = e^{\lambda x} \sin(\mu x)$  and  $y_2 = e^{\lambda x} \cos(\mu x)$  are linearly independent by using the definition.
- (c) Let two functions be defined as follows:

$$y_1(x) = \mathbb{1}_{[0,1]}(x) = \begin{cases} 1, & \text{when } 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad y_2(x) = \mathbb{1}_{[2,3]}(x) = \begin{cases} 1, & \text{when } 2 \leq x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

Show that the two functions are linearly independent.