Problem Set 7

Differential Equations

Fall 2025

Second order differential equations are very similar to higher order differential equations. While you have also learned about the theory as well as non-homogeneous cases for second order differential equation, we will get to understand problems in this part. Meanwhile, we will also get ourselves ready for higher order differential equations.

Clubs & Orgs Bulletin

Promote your club! https://forms.gle/V19BipzLyuAaWMyz8

Violet Patient Volunteering Program: Violet is a fantastic opportunity for clinical volunteering, peer education, and community engagement. We spread sexual and reproductive health education across 3 clinics in Baltimore, and our applications for our Spring 2026 volunteering program are open! The written application is due Sunday, October 26th at 11:59 pm. Check our instagram `@violet4teens` for the link :)

Society of Women Engineers: SWE is hosting a Professor Panel dinner on 10/22 from 67:30pm! Meet female WSE professors, learn about their research and paths into academia, and get advice on post-college opportunities. Registration on HopkinsGroups is required and food is provided. Follow us on Instagram `@jhuswe` to learn more!

Tip of the Week

The beginning of fall also means the start of flu season–get your shot today! Hopkins holds flu clinics on all of its campuses. Students must upload verification of their immunization or a valid exemption by Friday, Nov 21st, 2025. Find out more information here: https://wellbeing.jhu.edu/PrimaryCare/annual-flu-vaccine-requirement/.

1. (A Symmetric Solution). Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \sin^2(1-x)y = \cosh(x-1), \\ y(1) = e,\ \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.

Hint: Consider the interval in which the solution is unique. Note that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

2. (Non-homogeneous Differential Equations). Solve the following differential equations.

   (a) $y'' + 4y = t^2 + 3e^t$.

   (b) $y'' + 2y' + y = \dfrac{e^{-x}}{x}$.

3. (Repeated Roots and Wronskian). Let a differential equation of $y := y(x)$ be:

$$y''' + 3y'' + 3y' + y = 0.$$

Find the general solution the differential equation and give the Wronskian of your set of solutions.

4. (Higher order IVP with Dirichlet Condition). Consider the following initial value problem:

$$\begin{cases} 2y''' - 11y'' + 17y' - 6y = 0, \\ y(0) = 3, y(\log(4)) = 82, y(\log(9)) = 813. \end{cases}$$

Find the specific solution to the IVP.