PILOT

Problem Set 8

Differential Equations

Fall 2025

Up to right now, we have completed our expenditure of higher order differential equations. You should be familiar with the following concepts:

- Concepts:
  - Set of Solutions
  - Linear Independence
  - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
  - Characteristic Equation
  - Euler's Formula
  - Undetermined Coefficients
  - Reduction of Order
  - Variation of Parameters

Now, as we step into more linear algebra, we are going to review the key contents of this part of the class, and explore how are they related to linear systems.

Tip of the Week

Love thrifting, crafts, and good food? The Bmore Flea Market is open every Saturday until December from 10am-3pm at Broadway Market in Fells Point. Check out their Instagram `@bmoreflea` *for more info!*

1. (Finding General Solutions). Find the general solution to the following differential equation.

   (a) $$y'''(x) - 6y''(x) + 11y'(x) - 6y(x) = 0.$$

   (b) $$y''' = -y.$$

2. (Non-homogeneous Solutions). Find the general solution to the following differential equations:

   (a) $$y''' - 4y' = e^{-2t}.$$

   (b) $$y'' + 36y = e^{t}\sin(6t).$$

3. (Non-homogeneous Cases of Higher Order ODEs). Let a third order differential equation be as follows:

$$\ell[y(t)] = y^{(3)}(t) + 3y''(t) + 3y'(t) + y(t).$$

Let $\ell[y(t)] = 0$ be trivial initially.

(a) Find the set of all linearly independent solutions.

Then, assume that $\ell[y(t)]$ is non-trivial.

(b) Find the particular solution to $\ell[y(t)] = \sin t$.

(c) Find the particular solution to $\ell[y(t)] = e^{-t}$.

(d) Suppose that $\ell[y_1(t)] = f(t)$ and $\ell[y_2(t)] = g(t)$ where $f(t)$ and $g(t)$ are "good" functions. Find an expression to $y_3(t)$ such that $\ell[y_3(t)] = f(t) + g(t)$.

4. (Solving Linear Systems). Let $\mathbf{x} \in \mathbb{R}^2$, find the general solution of $\mathbf{x}$ for:

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \cdot \mathbf{x}.$$