PILOT

Problem Set 9

Differential Equations

Fall 2025

We will start to explore a new class of problems, the linear systems. First of all, you should be familiar with the real eigenvalue cases and building up the connections with higher order differential equations. At the same time, be aware of linear algebra as a potential tool for you to make a lot of the problems easier.

Tip of the Week

The Red Zone refers to the period of time between the start of fall semester and Thanksgiving when the majority of sexual assaults on campus occur. The peak in the Red Zone occurs around Halloween, so it is important to refresh your knowledge on consent and sexual assault resources during this time.
Confidential sexual assault resources at JHU include the Counseling Center Sexual Assault Helpline at (410) 516-7333 and the Hopkins Sexual Assault Resource Unit Hotline at (410) 516-7887.
Access more resources and educational materials on consent on Instagram `@jhusaru` and `@chewatjhu`.

1. (Linear System versus Second Order). Let an initial value problem for linear system on $x_1 := x_1(t)$ and $x_2 := x_2(t)$ be defined as follows:

$$\begin{cases} x_1' = 3x_1 - 2x_2, & x_1(0) = 3, \\ x_2' = 2x_1 - 2x_2, & x_2(0) = \dfrac{1}{2}. \end{cases}$$

   (a) Solve for the *general solution* for the linear system by considering $\mathbf{x} = (x_1, x_2)$.

   (b) Transform the *general system* into a single equation of second order. Then solve the second-order equation. Eventually, convert your solution of one variable back to the *general solution* to $x_1(t)$ and $x_2(t)$.

   (c) Find the particular solution using the initial conditions, then graph the parameterized curve on a $x_1 x_2$-plane with $t \geq 0$.

2. (Euler's Formula for Higher Order ODEs). Give the general solution to the following higher order differential equations:

$$y^{(6)} - 2y''' + y = 0.$$

3. (Directional Field for Linear System). For the following systems with $\mathbf{x} = (x_1, x_2)$, draw a direction field and plot some trajectories to characterize the solutions.

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \cdot \mathbf{x}.$$

4. (The "Big Guys"). Let $\mathbf{x} = (x_1, x_2)$ satisfy the following differential equations:

(a)
$$\mathbf{x}' = \begin{pmatrix} \frac{1}{42} & \frac{1}{21} \\ \frac{1}{14} & \frac{1}{21} \end{pmatrix} \cdot \mathbf{x}.$$

(b)
$$\mathbf{x}' = \begin{pmatrix} \frac{43}{42} & \frac{1}{21} \\ \frac{1}{14} & \frac{22}{21} \end{pmatrix} \cdot \mathbf{x}.$$

Solve the given linear systems.

Hint: Think about the *geometric* interpretation of eigenvalues and eigenvectors and try to *simplify* the matrix. (*Otherwise, the computation is hard.*)