Quiz 1 Review Set

Differential Equations

Fall 2025

1. Find the general solution for $y = y(t)$:

$$y' + 3y = t + e^{-2t},$$

then, describe the behavior of the solution as $t \to \infty$.

2. An autonomous differential equation is given as follows:
$$\frac{dy}{dt} = 4y^3 - 12y^2 + 9y - 2 \text{ where } t \geq 0 \text{ and } y \geq 0.$$
Draw a phase portrait and sketch a few solutions with different initial conditions.

3. For the first-order autonomous ODE:
$$\frac{dy}{dt} = \sin y + C,$$
where $C \in \mathbb{R}$ is a parameter. Determine any and all bifurcation values for the parameter C and sketch a bifurcation diagram.

4. Let an initial value problem be defined as follows:

$$\begin{cases} \left(12x^4 + 5x^2 + 6\right)\dfrac{dy}{dx} - \left(x^2\sin(x) + x^3\right)y = 0, \\ y(0) = 1. \end{cases}$$

Show that the solution to the above initial value problem is symmetric about $x = 0$.