Quiz 2 Review Set

Differential Equations

Fall 2025

1. Determine if the following differential equation is exact. If not, find the integrating factor to make it exact. (Hint: You can use the integrating factor from a canonical integrating factor problem). Then, solve for its general solution using the exactness method:

$$y'(x) = e^{2x} + y(x) - 1.$$

2. For the first-order autonomous ODE:

$$\frac{dy}{dt} = y^3 + 3y^2 + C,$$

where $C \in \mathbb{R}$ is a parameter. Determine any and all bifurcation values for the parameter C and sketch a bifurcation diagram.

3. Solve the following second order differential equations for $y = y(x)$:
   (a) $y'' + y' - 132y = 0.$
   (b) $y'' - 4y' = -4y.$
   (c) $y'' - 2y' + 3y = 0.$

4. Given a differential equation for $y = y(t)$ being:

$$t^3y'' + ty' - y = 0.$$

   (a) Verify that $y_1(t) = t$ is a solution to the differential equation.

   (b) Find the full set of solutions using reduction of order.

   (c) Show that the set of solutions from part (b) is linearly independent.