PILOT

Quiz 3 Review Set

Differential Equations

Fall 2025

1. Solve the general solution for $y = y(t)$ to the following second order non-homogeneous ODEs.

   (a) $$y'' + 2y' + y = e^{-t}.$$

   (b) $$y'' + y = \tan t.$$

2. Solve for the general solution to the following higher order ODE.

   (a)
   $$4\frac{d^4y}{dx^4} - 24\frac{d^3y}{dx^3} + 45\frac{d^2y}{dx^2} - 29\frac{dy}{dx} + 6y = 0.$$

   (b)
   $$\frac{d^4y}{dx^4} + y = 0.$$

   Hint: Consider the 8-th root of unity, *i.e.*, ζ_8, and verify which roots satisfies the polynomial.

3. Given the following second order initial value problem:
$$\begin{cases} \dfrac{d^2y}{dx^2} + \cos(1-x)y = x^2 - 2x + 1, \\ y(1) = 1, \qquad \dfrac{dy}{dx}(1) = 0. \end{cases}$$
Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.
Hint: Consider the interval in which the solution is unique.

4. Let a system of differential equations of $x_i(t)$ be as follows:

$$\begin{cases} x_1' = 3x_1 + 2x_2, & x_1(1) = 0, \\ x_2' = x_1 + 4x_2, & x_2(1) = 2. \end{cases}$$

   (a) Solve for the solution to the initial value problem.

   (b) Identify and describe the stability at equilibrium(s).