Quiz 4 Review Set

Differential Equations

Fall 2025

1. Let systems of differential equations be defined as follows, find the general solutions to the equations:

   (a) $$\mathbf{x}' = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x} = (x_1, x_2).$$

   (b) $$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}, \qquad \mathbf{x} = (x_1, x_2).$$

   (c) $$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 4 \\ 1 & 1 & 3 \\ 0 & 4 & 1 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x} = (x_1, x_2, x_3).$$

2. Solve the following initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

3. For the following non-linear systems, find all equilibrium(s) and classify their stability locally if they are locally linear.

(a)
$$\begin{cases} \dfrac{dx}{dt} = x - y^2, \\ \dfrac{dy}{dt} = x + x^2 - 2y. \end{cases}$$

(b)
$$\begin{cases} \dfrac{dx}{dt} = 2x + 3y^2, \\ \dfrac{dy}{dt} = x + 4y^2. \end{cases}$$

4. Let the following systems of (x, y) be functions of variable t:

   (a)
   $$\begin{cases} x' = (1 + x)\sin y, \\ y' = 1 - x - \cos y. \end{cases}$$

   (b)
   $$\begin{cases} x' = x - y, \\ y' = x - 2y + x^2. \end{cases}$$

   Identify the corresponding linear system, then evaluate the stability for the equilibrium at $(0, 0)$ by showing it is locally linear.