



Problem Set 9

Differential Equations

Spring 2025

As of right now, we have completed our expenditure of higher order differential equations. You should be familiar with the following concepts:

- Concepts:
 - Set of Solutions
 - Linear Independence
 - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
 - Characteristic Equation
 - Euler's Formula
 - Undetermined Coefficients
 - Reduction of Order
 - Variation of Parameters

Now, as we step into more linear algebra, we are going to review the key contents of this part of the class.

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Tip of the Week

Fall 2025 registration is coming up soon! Make sure to consult with your academic and faculty advisors, check the e-catalog, and speak with upperclassmen for course recommendations. This year's registration dates are April 7 for rising seniors, April 9 for rising juniors, and April 11 for rising sophomores.

1. (Reduction of Order or Integrating Method). Let a differential equation be:

$$y''(t) + \frac{2}{t}y'(t) = 0.$$

- (a) Verify that $y(t) = 1/t$ is one solution, then find a full set of solution.
- (b) Consider $\omega(t) = y'(t)$, solve the differential equation by using integrating factor.
- (c) Verify that the two methods give you the same set of the solutions.

2. (Non-homogeneous Cases of Higher Order ODEs). Let a third order differential equation be as follows:

$$\ell[y(t)] = y^{(3)}(t) + 3y''(t) + 3y'(t) + y(t).$$

Let $\ell[y(t)] = 0$ be trivial initially.

- (a) Find the set of all linearly independent solutions.

Then, assume that $\ell[y(t)]$ is non-trivial.

- (b) Find the particular solution to $\ell[y(t)] = \sin t$.

- (c) Find the particular solution to $\ell[y(t)] = e^{-t}$.

- (d)* Suppose that $\ell[y_1(t)] = f(t)$ and $\ell[y_2(t)] = g(t)$ where $f(t)$ and $g(t)$ are “good” functions. Find an expression to $y_3(t)$ such that $\ell[y_3(t)] = f(t) + g(t)$.

3. (Non-homogeneous Differential Equations). Solve the following differential equations.

(a) $y'' + 4y = t^2 + 3e^t.$

(b) $y'' + 2y' + y = \frac{e^{-x}}{x}.$

4. (Warm up in Linear Algebra). This problem reviews the basic concepts linear algebra concepts.

(a) Which of the following set of vectors are linearly independent in \mathbb{R} -vector space, what about \mathbb{C} -vector space? Justify your answer.

(i) $\alpha = \{(1, 1, 0), (0, 1, 1), (1, 0, 1)\},$

(ii) $\beta = \{(0, 1), (2, 3), (4, 5)\},$

(iii) $\gamma = \{1, i\}.$

(b) Let $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$ and $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$, compute the following:

(i) $A - 2B,$

(ii) $BA,$

(iii) $B^{-1}.$