



Problem Set 3
Differential Equations
Spring 2025

Welcome back to more ODEs. As we are familiarizing ourselves with the techniques of solving ODEs, let's see how these methods can be applied into various system, of how we can model some activities. At the same time, we shall also evaluate the behavior of ODEs without solving them.

Clubs & Orgs Bulletin

Promote your club! <https://forms.gle/V19BipzLyuAaWMyz8>

Bands@JHU: *Do you play a band instrument and want to be involved in an ensemble? Bands@JHU is the place to be! Rehearsals are Sunday evenings for the wind ensemble and Thursday evenings for the jazz bands. We welcome all musicians wanting to make some good music, so check us out at <https://tinyurl.com/ycz8zupt>*

Tip of the Week

Still looking for off-campus housing for the summer or fall? Check out the Johns Hopkins Off-Campus website which offers resources like apartment listings and roommate matches: <https://offcampushousing.jhu.edu/>. Make sure to talk with upperclassmen to learn more about options, pricing, and management experiences before you sign a lease.

1. (Stability of Autonomous ODEs). Draw the phase line and determine the stability of each equilibrium for the following autonomous differential equations:

(a) $y' = y^4 - 3y^3 + 2y^2.$

(b) $y' = y^{2025} - 1.$

(c) $y' = y^2 + 2y + C,$ where $C \in \mathbb{R}$ is a constant.

For part (c), determine the bifurcation values for the parameter C and sketch a bifurcation diagram.

2. (Integrating Factor & Exactness). Let a differential equation be defined as follows:

$$\frac{dy}{dx} = e^{2x} + y - 1.$$

- (a) What is the integrating factor ($\mu(x)$) for the equation? Solve for the general solution.
- (b) Is the equation *exact*? If not, make it exact, then find the general solution.
- (c) Do solutions from part (a) and (b) agree?

3. (Decay and Dating). Carbon-14, a radioactive isotope of carbon, is an effective tool in dating the age of organic compounds, as it decays with a relatively long period. Let $Q(t)$ denote the amount of carbon-14 at time t , we suppose that the decay of $Q(t)$ satisfies the following differential equation:

$$\frac{dQ}{dt} = -\lambda Q \text{ where } \lambda \text{ is the rate of decay constant.}$$

- (a) Let the half-life of carbon-14 be τ , find the rate of decay, λ .
- (b) Suppose that a piece of remain is discovered to have 10% of the original amount of carbon-14, find the age of the remain in terms of τ .

4. (End Behavior). Given following IVP:

$$\begin{cases} y' + \frac{1}{2}y = \sin t, \\ y(0) = 1. \end{cases}$$

- (a) Find the specific solution of the equation.
- (b) Describe the end behavior of the solution.