Problem Set 12

Differential Equations

Spring 2025

Up to now, we are about to finish the first order linear system. As we are about to get into the chapter, let us review what we have learned about linear systems.

- Concepts:
  - Vector space
  - Eigenspace
  - Existence & Uniqueness Theorem
- Methods to solve linear systems of ODEs:
  - Eigenvectors & Eigenvalues
  - Repeated Roots
  - Phase Portraits and Stability

Clubs & Orgs Bulletin

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There is no bulletin this week.

Tip of the Week

The Hopkins Food Pantry is a free resource for JHU affiliates facing food insecurity. Located at the LaB (right next to Homewood Apartments), it's open weekly on Mondays and Tuesdays to registered shoppers. Click here to learn more: https://tinyurl.com/jhu-pantry. (Note: The Hopkins Food Pantry does NOT conduct any background checks of your financial or conduct history. Any individual in the JHU community can access the pantry upon application approval. Shopper privacy is respected.)

1. (Directional Field for Linear System). For the following systems with $\mathbf{x} = (x_1, x_2)$, draw a direction field and plot some trajectories to characterize the solutions.

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \cdot \mathbf{x}.$$

2. (Complex Eigenvalue and Phase Portraits). Find the general solution and sketch a few phase portraits for:
$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \mathbf{x}.$$

3. (Phase Portraits for Repeated Roots). Find the solutions to the following linear system differential equation, sketch a few phase portraits, and classify its type and stability.

   (a)
$$\mathbf{x}' = \begin{pmatrix} 4 & 2 \\ -2 & 0 \end{pmatrix} \cdot \mathbf{x}.$$

   (b)
$$\mathbf{x}' = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \cdot \mathbf{x}.$$

4. (Critical Point). Find all the critical point in the following first order system:

$$\begin{cases} x' = 2x^3 - x^2 - 4x + 3 - y^2, \\ y' = 2x - y. \end{cases}$$