PILOT

Problem Set 1

Differential Equations

Spring 2025

Welcome to the PILOT Learning for `AS.110.302` Differential Equations and Applications. This course studies the *dynamics* of system(s), which are foundations of many mathematical models. While the PILOT program cultivates the mastering of knowledge, please also seek for comprehension through collaboration.

Prior to entering this class, let's remark on the key components that you should be familiar with:

- Basic Calculus:
  - Differentiation
  - Antiderivatives
  - Integration Techniques
- Sequences and Series:
  - Power Series
  - Convergence & Divergence
  - Taylor Expansion

Also, you should be familiar with arithmetics manipulation on polynomials and trigonometric functions.

Clubs & Orgs Bulletin

Promote your club! https://forms.gle/V19BipzLyuAaWMyz8
There is no bulletin this week.

Tip of the Week

The Johns Hopkins Ice Rink at Homewood is open for skating! It will operate from now through Sunday, February 23rd in front of the Imagine Center. The rink will have a variety of theme nights and serve beverages and snacks. To learn more and sign up for skating slots, visit www.jhu.edu/icerink.

1. (Review: Integration). As one of the most important skills of differential equations, the study requires proficiency in integration. By the *Fundamental Theorem of Calculus*, the basics of most calculations are on finding antiderivatives. Please evaluate the following indefinite integrals:

   (a) $$\int e^{1/x} \cdot \frac{1}{x^2} dx.$$

   (b) $$\int \sin(5x) e^{-x} dx.$$

   (c) $$\int \cos(2t) \tan(t) dt.$$

2. (Review: Series). As we dive into fundamentals of mathematics, it is inevitable to encounter *sequences* and their sums. Discuss about the following sequences if they converge or not. If they converge, find the explicit sum.

   (a) $$\sum_{k=1}^{\infty} \frac{1}{k}.$$

   (b) $$\sum_{k=0}^{\infty} \frac{1}{k!}.$$

   (c) $$\sum_{k=0}^{\infty} \frac{1}{(4k+1)!}.$$

3. (Direction Field). Let a differential equation be defined as follows:
$$\frac{dy}{dx} = y^3 - 7y^2 + 16y - 12 \text{ where } x \geq 0 \text{ and } y \geq 0.$$
Recall that a differential equation is:

- **ordinary** if it is composed of only ordinary derivatives ($d^n y/dx^n$ or $y^{(n)}$), and **partial** if it contains any partial derivatives ($\partial^n y/\partial x^n$ or $\partial_x^n y$);
- **linear** if the operator $L[y] = f(x)$ is linear with respect to y, and **nonlinear** otherwise.
- of **order** k if the highest order of derivative is k.

(a) The above differential equation is (ordinary / partial), (linear / nonlinear) differential equation of order ____.

(b) Sketch a direction field on the differential equation, and state the equilibriums of y, interpret their stability.

4. (Constructing ODE). Let $x(t) = t^2 e^t$.

   (a) Construct a second order ODE such that $x(t)$ is a solution and the differential equation includes all terms of $x(t)$, $x'(t)$ and $x''(t)$, along with (maybe) some leftover terms independent of x.
   Hint: Take the first and second derivative of $x(t)$ and fit them together into some linear combinations.

   (b) Are the ODEs satisfying (a) unique?