PILOT

Problem Set 7

Differential Equations

Spring 2025

While we move into more topics of high order differential equations, we are going to see how the characteristic equations, Wronskian, and similar concepts extends to higher order cases. In this set, we will practice on how to find solutions in different cases, attempting to reconstruct initial value problems, and understanding the theorems with examples.

Tip of the Week

If you ever experience a behavioral health crisis or are concerned about someone else, call and connect with JHU's Behavioral Health Crisis Support Team 24/7 at (410) 516-9355. Find out more information at https://wellbeing.jhu.edu/bhcst.

1. (LI Set of Solutions). Find the general solution to the following differential equation, and verify that your solution is a linearly independent set of solutions.
$$y'''(x) - 6y''(x) + 11y'(x) - 6y(x) = 0.$$

2. (Constructing Solutions, Again.) Construct an initial value problem with Cauchy conditions for the following solutions:

   (a) $$y(t) = 4e^{3t} - e^{-2t}.$$

   (b) $$y(t) = e^{2t}\cos t + e^{2t}\sin t + e^{2t}.$$

3. (Euler's Formula for Higher Order ODEs). Give the general solution to the following higher order differential equations:

$$y^{(6)} - 2y''' + y = 0.$$

4.* (A Symmetric Solution.) Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \sin^2(1-x)y = \cosh(x-1), \\ y(1) = e, \ \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.

Hint: Consider the interval in which the solution is unique.

Also, note that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.