Midterm 1 Review Problem Set

Differential Equations

Spring 2025

Instructions:

The set of questions serves as PILOT practices to midterm 1 for the Spring 2025 term of `AS.110.302` Differential Equations and Applications at Johns Hopkins University.

- The questions are designed to be done without notes and calculators.
- Solutions to selected questions will be discussed during the PILOT review session.

1. Find the general solution for $y = y(t)$:
$$y' + 3y = t + e^{-2t},$$
then, describe the behavior of the solution as $t \to \infty$.

2. Given an initial value problem:
$$\begin{cases} \dfrac{dy}{dt} - \dfrac{3}{2}y = 3t + 2e^t, \\ y(0) = y_0. \end{cases}$$
   (a) Find the integrating factor $\mu(t)$.
   (b) Solve for the particular solution for the initial value problem.
   (c) Discuss the behavior of the solution as $t \to \infty$ for different cases of y_0.

3. An autonomous differential equation is given as follows:
$$\frac{dy}{dt} = 4y^3 - 12y^2 + 9y - 2 \text{ where } t \geq 0 \text{ and } y \geq 0.$$
Draw a phase portrait and sketch a few solutions with different initial conditions.

4. Determine if the following differential equation is exact. If not, find the integrating factor to make it exact. Then, solve for its general solution:
$$y'(x) = e^{2x} + y(x) - 1.$$

5. For the first-order autonomous ODE:
$$\frac{dy}{dt} = \sin y + C,$$
where $C \in \mathbb{R}$ is a parameter. Determine any and all bifurcation values for the parameter C and sketch a bifurcation diagram.

6. Let an initial value problem be defined as follows:
$$\begin{cases} \left(12x^4 + 5x^2 + 6\right)\dfrac{dy}{dx} - \left(x^2 \sin(x) + x^3\right)y = 0, \\ y(0) = 1. \end{cases}$$
Show that the solution to the above initial value problem is symmetric about $x = 0$.