PILOT Activity Slides

Differential Equations

Johns Hopkins University

Spring 2025

Welcome to ODE PILOT Session!

PILOT Sessions varies for each PILOT leader, please check in with your leader.

Session Information

- Term: Spring 2025
- Meeting Schedule:
  - Dates: Between January 21st and April 24th (inclusive).
  - Day of the Week: _____.
  - Except for Spring break (March 17th - March 21st).
  - Time: _____, Eastern Time.
  - Location: _____.
- Leader: _____ (email: _____).

Resources for PILOT

PILOT Webpage for ODEs

`https://jhu-ode-pilot.github.io/SP25/`

Please save the page as an resource for this course.

- The weekly problem sets will be updated on the webpage throughout the semester.
  The solution is **password encrypted**, so please join the sessions.
- The page contains general resources and announcements for PILOT as well.

Additional Resources

Review sessions will be planned prior to exams. Review sets will be published before prior week.

- The review problem sets and review session information will be updated on the webpage.

Exam Number	Exam Date	Check Updates By
Midterm 1	February 19th	February 9th
Midterm 2	April 2nd	March 23rd
Final	May 9th	April 27th

If you are looking for more conceptual challenge, there is an **Additional Problem Set** on the webpage that you can try to consolidate your understanding on the course.

Ground Expectations

In participating the PILOT program, you are expected to:

- Present to the weekly meeting. If you have any time conflicts or reasons, please notify your PILOT leader. Note that attendance will be taken, and multiple absences might result in removal from this session.
- Discuss with other students and/or the PILOT leader during meetings, while you may propose any questions and/or concerns if you have any.

Please be respectful and polite to other students!

If you found any of the contents a mental challenge or uncomfortable, please contact your leader or contact the Director of PILOT at Jenna Hoffman (`jhoffm71@jhu.edu`).

Introducing yourselves

Let's get to know each other.

Introduction Questions

This section aims to help you introduce yourselves to the other students, please use a few minutes to think about the problems and introduce yourselves to your peers.

Think about yourself. Get ready to introduce yourself by addressing the following information:

- Your name,
- Your expected graduation year,
- Your major(s) and minor(s),
- Your interested area(s) in mathematics.

Outreach Problems

ODEs are useful tools at many places.

Here are some open questions

If you do not prefer tedious introductions, choose one of the following questions and give a creative answer.

ODEs Outreach

What is one thing in your life, that you imagine ODEs can model. Explain why?

Examples

- How will the population of _____ change over time.
- How will a _____ dynamical system look like.
- Can I model _____.
- . . .

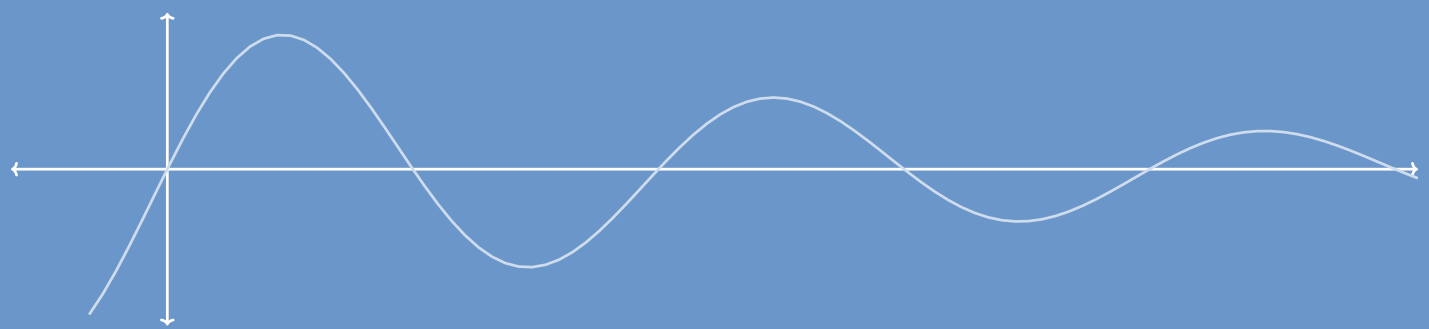
ODEs Outreach

Use the function:

$$f(x) = 10 \sin\left(\frac{x}{5}\right) \exp\left(-\frac{x}{50}\right)$$

to describe something.

The graph of the above function.



ODEs Outreach

If you can define a mathematical constant, what would you define?

Example mathematical constants.

- $\pi, \tau, \cdots$.
- $e = \sum_{n=0}^{\infty} \frac{1}{n!}$.
- $\cdots$

ODEs Outreach

Do you have a favorite theorem/formula/kernel?

Kernel

A good kernel $K_\delta(x)$ should be integrable (on $\mathbb{R}$) and satisfies the following for all $\delta > 0$:

1. $\int_{\mathbb{R}^d} K_\delta(x)dx = 1$,
2. $\int_{\mathbb{R}^d} |K_\delta(x)|dx \leq A$, and
3. for every $\eta > 0$, $\int_{|x|\geq\eta} |K_\delta(x)|dx \to 0$ as $\delta \to 0$,

where A is a constant depending on δ.

ODEs Outreach

Weierstrass Approximate Theorem guarantees uniform convergence for continuous functions, whereas *Fourier Convergence Theorem* only guarantees convergence for square integrable functions. Can you think of some places where you find trade-off situations?

ODEs Outreach

In mathematics, we call a question *well-posed* if it aligns with the following properties:

1. Existence: There exists at least one solution;
2. Uniqueness: There exists at most one solution;
3. Continuity: The solution depends continuously on the data, *i.e.*, a small error on initial/boundary data entails a small error on the solution.

Can you think of any "well-posed" questions?

Ordering Game

Ordered Sets

The field of real numbers is ordered. Thus, each person can select a number, and thus determining an order for the group.

Of course, a partial order $\prec$ must satisfy a few rules, and there are many other orders.

Dictionary Order

Of course, there are different ordering methods. For examples, you can look up *dictionary order* for complex numbers.

Ordering Game

Below are subsets of real numbers, select a number from a set:

- $\left\{0, 1, 2, -1, \frac{1}{2}, \sqrt{2}, \pi, e\right\}$,
- $\left\{\sin \frac{2k\pi}{15} : k \in \mathbb{Z} \wedge 0 \leq k \leq 14\right\}$,
- $\mathbb{R} \setminus \mathbb{Q}$ (irrational numbers),
- $\left\{\det \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \det \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix}, \det \begin{pmatrix} 1 & 3 \\ 4 & 7 \end{pmatrix}, \det \begin{pmatrix} 2 & 0 \\ 0 & 7 \end{pmatrix}, \det \begin{pmatrix} 1 & 0 \\ 4 & 3 \end{pmatrix}, \det \begin{pmatrix} 1 & -2 \\ 12 & 13 \end{pmatrix}\right\}$,

Ordering Game (Cont.)

- $\mathbb{A} \cap \mathbb{R}$ (real, algebraic number),
- $\{f(-10), f(-2), f(0), f(3), f(5), f(20)\}$, where $f(x) = \int_0^\infty e^{-xt} \sin t dt$,
- $\mathbb{Q}(\sqrt{2}, \sqrt{3}) := \left\{a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6} : a, b, c, d \in \mathbb{Q}\right\}$,
- $\{n$: regular n-gon is constructible$\}$, *Hint:* Regular n-gon is constructible $\iff$ $\phi(n)$ is an integral power of 2,
- $\{F_n\}_n$ (Fibonacci sequence).