

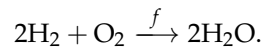


Final Review Problem Set

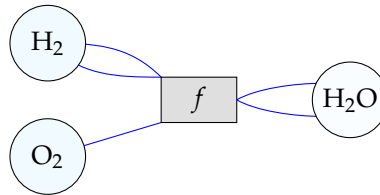
Differential Equations

Spring 2026

1. Consider the following chemical equation of hydrogen gas combustion in oxygen gas:



We may represent it from a graphical representation.



Assume that the reaction rate is constant $\kappa := \text{rate}(f)$. Construct the nonlinear system of the concentration of H_2 and O_2 , sketch a few trajectories for different initial conditions for different starting concentrations.

2. Carbon-14, a radioactive isotope of carbon, is an effective tool in dating the age of organic compounds, as it decays with a relatively long period. Let $Q(t)$ denote the amount of carbon-14 at time t , we suppose that the decay of $Q(t)$ satisfies the following differential equation:

$$\frac{dQ}{dt} = -\lambda Q \text{ where } \lambda \text{ is the rate of decay constant.}$$

- (a) Let the half-life of carbon-14 be τ , find the rate of decay, λ .
- (b) Suppose that a piece of remain is discovered to have 10% of the original amount of carbon-14, find the age of the remain in terms of τ .

3. Find the solution to the following ODE:

$$\frac{y'}{x} + e^{x^2} y = e^{x^2}.$$

4. Consider the Hamiltonian potential as:

$$H(x, y) = x^2y + \frac{y^3}{3} + \sin x.$$

Construct the nonlinear system as:

$$\begin{cases} x' = \frac{\partial H}{\partial y}, \\ y' = -\frac{\partial H}{\partial x}. \end{cases}$$

Find the trajectories in finding a relationship between x and y (as level curves).

5. Find the solution to the following higher order ODE:

$$y''' - 6y'' + 12y' - 9y = 0.$$

6. Solve the following nonhomogeneous ODE:

$$(x^2 + 1)y'' - 2xy' + 2y = x.$$

Hint: Observe a homogeneous solution, then reduction of order, eventually variation of parameters.

7. For the following non-linear systems, find all equilibrium(s) and classify their stability locally if they are locally linear.

(a)
$$\begin{cases} \frac{dx}{dt} = x - y^2, \\ \frac{dy}{dt} = x + x^2 - 2y. \end{cases}$$

(b)
$$\begin{cases} \frac{dx}{dt} = 2x + 3y^2, \\ \frac{dy}{dt} = x + 4y^2. \end{cases}$$

8. Let the following systems of (x, y) be functions of variable t :

(a)
$$\begin{cases} x' = (1 + x) \sin y, \\ y' = 1 - x - \cos y. \end{cases}$$

(b)
$$\begin{cases} x' = x - y, \\ y' = x - 2y + x^2. \end{cases}$$

Identify the corresponding linear system, then evaluate the stability for the equilibrium at $(0, 0)$ by showing it is locally linear.