Midterm 1 Review Problem Set

Differential Equations

Spring 2026

1. Find the general solution for $y = y(t)$:

$$y' + 3y = t + e^{-2t},$$

then, describe the behavior of the solution as $t \to \infty$.

2. An autonomous differential equation is given as follows:

$$\frac{dy}{dt} = 4y^3 - 12y^2 + 9y - 2 \text{ where } t \geq 0 \text{ and } y \geq 0.$$

Draw a phase portrait and sketch a few solutions with different initial conditions.

3. Let an initial value problem be defined as follows:

$$\begin{cases} \left(12x^4 + 5x^2 + 6\right)\dfrac{dy}{dx} - \left(x^2 \sin(x) + x^3\right)y = 0, \\ y(0) = 1. \end{cases}$$

Show that the solution to the above initial value problem is symmetric about $x = 0$.

4. Determine if the following differential equation is exact. If not, find the integrating factor to make it exact. (Hint: You can use the integrating factor from a canonical integrating factor problem). Then, solve for its general solution using the exactness method:
$$y'(x) = e^{2x} + y(x) - 1.$$

5. Solve the following second order differential equations for $y = y(x)$:

   (a) $$y'' + y' - 132y = 0.$$

   (b) $$y'' - 4y' = -4y.$$

   (c) $$y'' - 2y' + 3y = 0.$$

6. Given a differential equation for $y = y(t)$ being:

$$t^3 y'' + ty' - y = 0.$$

   (a) Verify that $y_1(t) = t$ is a solution to the differential equation.

   (b) Find the full set of solutions using reduction of order.

   (c) Show that the set of solutions from part (b) is linearly independent.