PILOT

# Midterm 2 Review Problem Set

Differential Equations

Spring 2026

1. Solve the general solution for $y = y(t)$ to the following second order non-homogeneous ODEs.

   (a) $$y'' + 2y' + y = e^{-t}.$$

   (b) $$y'' + y = \tan t.$$

2. Solve for the general solution to the following higher order ODE.

   (a)
   $$4\frac{d^4y}{dx^4} - 24\frac{d^3y}{dx^3} + 45\frac{d^2y}{dx^2} - 29\frac{dy}{dx} + 6y = 0.$$

   (b)
   $$\frac{d^4y}{dx^4} + y = 0.$$

   *Hint:* Consider the 8-th root of unity, *i.e.*, $\zeta_8$, and verify which roots satisfies the polynomial.

3. Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \cos(1-x)y = x^2 - 2x + 1, \\ y(1) = 1, \qquad \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.

*Hint:* Consider the interval in which the solution is unique.

4. Let a system of differential equations of $x_i(t)$ be as follows:

$$\begin{cases} x_1' = 3x_1 + 2x_2, & x_1(1) = 0, \\ x_2' = x_1 + 4x_2, & x_2(1) = 2. \end{cases}$$

   (a) Solve for the solution to the initial value problem.

   (b) Identify and describe the stability at equilibrium(s).

5. Let systems of differential equations be defined as follows, find the general solutions to the equations:

(a)
$$\mathbf{x}' = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x} = (x_1, x_2).$$

(b)
$$\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}, \qquad \mathbf{x} = (x_1, x_2).$$

(c)
$$\mathbf{x}' = \begin{pmatrix} 1 & 0 & 4 \\ 1 & 1 & 3 \\ 0 & 4 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x} = (x_1, x_2, x_3).$$

6. Solve the following initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

Plot the phase portrait for the general problem, note the trajectory for the initial value.