Problem Set 1

Differential Equations

Spring 2026

Welcome to the PILOT Learning for `AS.110.302` Differential Equations and Applications. This course studies the *dynamics* of system(s), described as ordinary differential equations, which are foundations of many more advanced mathematical models. While the PILOT program cultivates the mastering of knowledge, please also seek for comprehension through collaboration.

Prior to entering this class, let's remark on the key components that you might be familiar with:

- Pre-Calculus:
  - Polynomials and roots
  - Trigonometry
  - Complex Numbers
- Basic Calculus:
  - Differentiation
  - Antiderivatives
  - Integration Techniques
- Sequences and Series:
  - Power Series
  - Convergence & Divergence
  - Taylor Expansion
- Basic Linear Algebra Concepts:
  - Linear Independence
  - Eigenspace (optional)
  - Determinants

Tip of the Week

Important registration dates this semester:

The Johns Hopkins Ice Rink at Homewood is open for skating! It will operate from now through February 22, 2026 at the same location as last year: just off West University Parkway in the Imagine Center parking lot. The rink has a variety of theme nights, including Disco, ugly sweater, and senior night. To learn more and sign up for skating slots, visit https://www.jhu.edu/johns-hopkins-ice-rink-at-homewood/.

1. (Indefinite Integrals). As one of the most important skills of differential equations, the study requires proficiency in integration. By the *Fundamental Theorem of Calculus*, the basics of most computations are on finding antiderivatives. Please evaluate the following indefinite integrals:

   (a)
   $$\int e^{1/x} \cdot \frac{1}{x^2} dx.$$

   (b)
   $$\int \sin(5x) e^{-x} dx.$$

   (c)
   $$\int \cos(2t) \tan(t) dt.$$

2. (Separable ODE.) Solve the following initial value problem (IVP) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = (x \log x)^{-1}, \\ y(e) = -6. \end{cases}$$

Note: Here $\log(x) := \log_e(x)$ is the natural logarithm function, which may be written as $\ln(x)$.

3. (Direction Field). Let a differential equation be defined as follows:

$$\frac{dy}{dx} = y^3 - 7y^2 + 16y - 12 \text{ where } x \geq 0 \text{ and } y \geq 0.$$

Recall that a differential equation is:

- **ordinary** if it is composed of only ordinary derivatives ($d^n y/dx^n$ or $y^{(n)}$), and **partial** if it contains any partial derivatives ($\partial^n y/\partial x^n$ or $\partial_x^n y$);
- **linear** if the operator $L[y] = f(x)$ is linear with respect to y, and **nonlinear** otherwise.
- of **order** k if the highest order of derivative is k.

(a) The above differential equation is (ordinary / partial), (linear / nonlinear) differential equation of order ____.

(b) Sketch a direction field on the differential equation, and state the equilibriums of y, interpret their stability.

4. (Constructing ODEs from Solutions). Let $x(t) = t^2 e^t$.
   (a) Construct a second order ODE such that $x(t)$ is a solution and the differential equation includes all terms of $x(t)$, $x'(t)$ and $x''(t)$, along with some leftover terms independent of x.
   Hint: Take the derivative of $x(t)$ and fit them together into some linear combinations.
   (b) Are the ODEs satisfying (a) unique?