PILOT

Problem Set 10

Differential Equations

Spring 2026

As of right now, we have completed our expenditure of higher order differential equations. You should be familiar with the following concepts:

- Concepts:
  - Set of Solutions
  - Linear Independence
  - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
  - Characteristic Equation
  - Euler's Formula
  - Undetermined Coefficients

Now, as we step into linear systems, we will also work on the concepts of linear algebra, namely the eigenvalues and eigenvector.

Tip of the Week

There are multiple modes of free transportation to navigate around Baltimore. Use the TransLoc app to book the Blue Jay Shuttle, which runs between 6pm and 2am. The Collegetown Shuttle can be taken to Towson Center, and the Charm City Circulator stops at Mount Vernon and Penn Station. The JHMI shuttle is often used to reach the Johns Hopkins Hospital. Visit Homewood Student Affairs: Resources for Students for more information.

1. (Solving Linear Systems). Let $\mathbf{x} \in \mathbb{R}^2$, find the general solution of $\mathbf{x}$ for:

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \cdot \mathbf{x}.$$

2. (Linear System versus Second Order). Let an initial value problem for linear system on $x_1 := x_1(t)$ and $x_2 := x_2(t)$ be defined as follows:

$$\begin{cases} x_1' = 3x_1 - 2x_2, & x_1(0) = 3, \\ x_2' = 2x_1 - 2x_2, & x_2(0) = \dfrac{1}{2}. \end{cases}$$

(a) Solve for the *general solution* for the linear system by considering $\mathbf{x} = (x_1, x_2)$.

(b) Transform the *general system* into a single equation of second order. Then solve the second-order equation. Eventually, convert your solution of one variable back to the *general solution* to $x_1(t)$ and $x_2(t)$.

(c) Find the particular solution using the initial conditions, then graph the parameterized curve on a x_1x_2-plane with $t \geq 0$.

3. (Euler's Formula for Higher Order ODEs). Give the general solution to the following higher order differential equations:

$$y^{(6)} - 2y''' + y = 0.$$

4. (The "Big Guys"). Let $\mathbf{x} = (x_1, x_2)$ satisfy the following differential equations:

(a)
$$\mathbf{x}' = \begin{pmatrix} \frac{1}{42} & \frac{1}{21} \\ \frac{1}{14} & \frac{1}{21} \end{pmatrix} \cdot \mathbf{x}.$$

(b)
$$\mathbf{x}' = \begin{pmatrix} \frac{43}{42} & \frac{1}{21} \\ \frac{1}{14} & \frac{22}{21} \end{pmatrix} \cdot \mathbf{x}.$$

Solve the given linear systems.

Hint: Think about the *geometric* interpretation of eigenvalues and eigenvectors and try to *simplify* the matrix. (*Otherwise, the computation is hard.*)