Problem Set 11

Differential Equations

Spring 2026

As we are getting deeper into linear system of first order ODEs, we are learning how to construct solutions to the cases where the eigenvalues could be complex, repeated, and we have different ways of solving each case. Moreover, by having a two dimensional system, we can consider the propagation of time as a "parametrized graph" so we can explore the trajectories of such system.
Hang on tight, we will have a lot of sketching this week.

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Tip of the Week

Did you know that you can access previous years course lists? Check out https://courses.jhu.edu/, which has information that can be helpful to plan your future schedule! Also, make sure to keep track of your coursework using Semesterly, at https://semester.ly/.

1. (Directional Field for Linear System). For the following systems with $\mathbf{x} = (x_1, x_2)$, draw a direction field and plot some trajectories to characterize the solutions.

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \cdot \mathbf{x}.$$

2. (Another "Big Guy"). Let $\mathbf{x} \in \mathbb{R}^4$, find the general solution of $\mathbf{x}$ for:

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 3 & 1 \end{pmatrix} \cdot \mathbf{x}.$$

3. (Zero Eigenvalue). Let a system of $\mathbf{x} = (x_1, x_2)$ be defined as:

$$\mathbf{x}' = \begin{pmatrix} -3 & -6 \\ 1 & 2 \end{pmatrix} \cdot \mathbf{x}.$$

   (a) Find the eigenvalues and eigenvectors for $\begin{pmatrix} -3 & -6 \\ 1 & 2 \end{pmatrix}$.

   (b) Give a full set of solutions to the differential equation. Plot some trajectory on the x_1x_2-plane.

   (c) Let A be an arbitrary square matrix. Show that A is non-invertible if and only if A has zero as an eigenvalue.
   Note: Please avoid using the definition that the determinant is the product of all eigenvalues. Moreover, consider the geometric implication of eigenvalue to account for invertibility.

4. (Complex Eigenvalue and Phase Portraits). Find the general solution and sketch a few phase portraits for the following problems:

(a)
$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \cdot \mathbf{x}.$$

(b)
$$\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \mathbf{x}.$$