PILOT

Problem Set 2

Differential Equations

Spring 2026

As we have dived into the contents of first order linear differential equations, you should have noticed many techniques, such as method of separation, integrating factors, or exactness. Meanwhile, we are about the see practice on these solving techniques and explore an explanation of such concept.

Tip of the Week

The Life Design Lab:

The Life Design Lab is a useful resource for career and personal development. Collaborate with Life Design Educators (LDEs) that correspond with your academic department on honing professional skills (resumes, CVs, interview skills), meet alumni, and learn how to find opportunities (internships and research) that prepare you for a life beyond Hopkins. This fall, the Life Design Lab is offering drop-in office hours that take place in person at the NEW Imagine Center (113 W. University Pkwy. Baltimore). No appointment needed! Learn more about LDL and drop-in hours at https://imagine.jhu.edu/channels/life-design-lab/.

1. (A Separable ODE). Solve the following initial value problems (IVPs) on $y = y(x)$, and specify the domain for your solution:

$$\begin{cases} y' = y(y+1), \\ y(0) = 1. \end{cases}$$

2. (Integrating Factor). Solve for the general solution to the following ODE with $y = y(t)$:

$$2y' + y = 3t.$$

3. (Linearity of Solutions). Let $y = y_1(t)$ be a solution to $y' + p(t)y = 0$, and let $y = y_2(t)$ be a solution to $y' + p(t)y = q(t)$.

   (a) Show that $y = y_1(t) + y_2(t)$ is then also a solution to $y' + p(t)y = q(t)$.

   (b) Give a general form of the solution to the differential equation $y' + p(t)y = q(t)$.
   Hint: Use some constant C and extend from part (a).

   (c) Give a general form of the solution to the differential equation $y' + p(t)y = Dq(t)$, where $D \in \mathbb{R}$ is a constant.

4. (Integrating Factor for IVP). Given an initial value problem:

$$\begin{cases} \frac{dy}{dt} - \frac{3}{2}y = 3t + 2e^t, \\ y(0) = y_0. \end{cases}$$

   (a) Find the integrating factor $\mu(t)$.

   (b) Solve for the particular solution for the initial value problem.

   (c) Discuss the behavior of the solution as $t \to \infty$ for different cases of y_0.