Problem Set 3: Solutions

Differential Equations

Spring 2026

Welcome back to more ODEs. As we are familiarizing ourselves with the techniques of solving ODEs, let's see how these methods can be applied into various system, of how we can model some activities. At the same time, we shall also evaluate the behavior of ODEs without solving them.

Tip of the Week

The Hopkins Food Pantry is a free resource for JHU affiliates facing food insecurity. Located at the LaB (right next to Homewood Apartments), it's open weekly on Mondays and Tuesdays to registered shoppers.
Learn more at https://studentaffairs.jhu.edu/student-life/student-outreach-support/hopkins-food-pantry/.

1. (Stability of Autonomous ODEs). Draw the phase line and determine the stability of each equilibrium for the following autonomous differential equations:

   (a) $$y' = y^4 - 3y^3 + 2y^2.$$

   (b) $$y' = y^{2026} - 1.$$

2. (More Integrating Factor). Solve for the general solution to the following ODEs with $y = y(t)$:
$$y' + \log(t)y = t^{-t}.$$

3. (More IVPs). Given following IVPs:

   (a)
$$\begin{cases} y' + \dfrac{1}{2}y = \sin t, \\ y(0) = 1. \end{cases}$$

   (b)
$$\begin{cases} y' = \dfrac{1}{x^4 - 1}, \\ y(0) = 0. \end{cases}$$

   Find the specific solution of the equation, note the domain of the solution, and describe the end behavior of the solution if it appears in the valid domain.

4. (Modeling with ODEs). Carbon-14, a radioactive isotope of carbon, is an effective tool in dating the age of organic compounds, as it decays with a relatively long period. Let $Q(t)$ denote the amount of carbon-14 at time t, we suppose that the decay of $Q(t)$ satisfies the following differential equation:

$$\frac{dQ}{dt} = -\lambda Q \text{ where } \lambda \text{ is the rate of decay constant.}$$

   (a) Let the half-life of carbon-14 be τ, find the rate of decay, λ.

   (b) Suppose that a piece of remain is discovered to have 10% of the original amount of carbon-14, find the age of the remain in terms of τ.