PILOT

Problem Set 5

Differential Equations

Spring 2026

As we wrap up the first main part of the class, let's remark on the key components so far about first order ordinary differential equations:

- Concepts:
  - Existence
  - Uniqueness
  - General Solutions
  - Specific Solutions (IVPs)
- Methods to solve first order ODEs:
  - Separable ODEs
  - Integrating Factor
  - Autonomous ODEs
  - Exact ODEs
- Behavior analyses:
  - Directional Field
  - End Behavior
  - Phase Line
  - Bifurcation Diagram

Get ready for more classes of differential equations!

Tip of the Week

If you need some extra help with an engineering class, JHU Study Consulting just added an option for express support. Express support is hosted by volunteers from Tau Beta Pi, the engineering honor society, and a good alternative for those who do not want a high-commitment weekly session. Direct booking is available on JHU Academic Supports website.

1. (Second Order Differential Equations). Find the general solution to the following second order differential equations on $y := y(t)$. Then verify using Wronskian that the solutions are linearly independent.

   (a) $$y'' - 3y' + 2y = 0.$$

   (b) $$y'' + 12y' - 3y = 0.$$

2. (Integrating Factor & Exactness). Let a differential equation be defined as follows:

$$\frac{dy}{dx} = e^{2x} + y - 1.$$

(a) What is the integrating factor ($\mu(x)$) for the equation? Solve for the general solution.

(b) Is the equation *exact*? If not, make it exact, then find the general solution.

(c) Do solutions from part (a) and (b) agree?

3. (Second Order IVP). Let an initial value problem for $y = y(t)$ be defined as follows:

$$\begin{cases} 4y'' - y = 0, \\ y(0) = 2,\ y'(0) = \beta, \end{cases}$$

where β is a real constant.

   (a) Find the specific solution to the initial value problem. Express your solution with constant β.

   (b) Find the value of β such that the solution *converges* to 0 as t tends to infinity.

4. (Euler's Theorem). In our study of differential equations, our main focus is on *real-valued functions*. But we are about to see complex numbers in our story. **Euler's theorem** states that for any $z \in \mathbb{C}$, we have:

$$\exp(\mathrm{i}z) = \cos(z) + \mathrm{i}\sin(z).$$

   (a) To review on complex numbers, compute/simplify the following expressions:

$$\text{(i)}(2+5\mathrm{i}) \times (1+2\mathrm{i}), \qquad \text{(ii)}\frac{2-3\mathrm{i}}{1+\mathrm{i}}, \qquad \text{(iii)}\overline{2+5\mathrm{i}}, \qquad \text{(iv)}(20+25\mathrm{i}) \times (\overline{20+25\mathrm{i}}).$$

   (b) Write the following complex exponentials in terms of a sum of the real and imaginary parts:

$$\text{(i)}\exp(\mathrm{i}), \qquad \text{(ii)}\exp\left(\frac{\pi\mathrm{i}}{3}\right), \qquad \text{(iii)}\exp(2+2\mathrm{i}).$$