PILOT

Problem Set 7: Solutions

Differential Equations

Spring 2026

By this time, we have completed our expenditure of the second order differential equations. You should be familiar with the following concepts:

- Concepts:
  - Space of Solutions
  - Linear Independence
  - Existence & Uniqueness Theorem
- Methods to solve higher order ODEs:
  - Characteristic Equation
  - Euler's Formula
  - Undetermined Coefficients
  - Reduction of Order
  - Variation of Parameters

Now, as we step into higher order differential equations, we will see natural extensions of these concepts into a higher order scenario.

Tip of the Week

The Life Design Lab:

The Life Design Lab is a useful resource for career and personal development. Collaborate with Life Design Educators (LDEs) that correspond with your academic department on honing professional skills (resumes, CVs, interview skills), meet alumni, and learn how to find opportunities (internships and research) that prepare you for a life beyond Hopkins. This fall, the Life Design Lab is offering drop-in office hours that take place in person at the NEW Imagine Center (113 W. University Pkwy. Baltimore). No appointment needed! Learn more about LDL and drop-in hours at https://imagine.jhu.edu/channels/life-design-lab/.

1. (Constructing IVPs, Again.) Construct an initial value problem with Cauchy conditions for the following solutions:

   (a) $$y(t) = 4e^{3t} - e^{-2t}.$$

   (b) $$y(t) = e^{2t}\cos t + e^{2t}\sin t.$$

2. (A Symmetric Solution). Given the following second order initial value problem:

$$\begin{cases} \dfrac{d^2y}{dx^2} + \sin^2(1-x)y = \cosh(x-1), \\ y(1) = e, \ \dfrac{dy}{dx}(1) = 0. \end{cases}$$

Prove that the solution $y(x)$ is symmetric about $x = 1$, *i.e.*, satisfying that $y(x) = y(2-x)$.

Hint: Consider the interval in which the solution is unique. Note that $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.

3. (Non-homogeneous Differential Equations). Solve the following differential equations.

   (a) $$y'' + 4y = t^2 + 3e^t.$$

   (b) $$y'' + 2y' + y = \frac{e^{-x}}{x}.$$

4. (Repeated Roots and Wronskian). Let a differential equation of $y := y(x)$ be:

$$y''' + 3y'' + 3y' + y = 0.$$

Find the general solution the differential equation and give the Wronskian of your set of solutions.